Polarized radiative transfer in discontinuous media

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Abstract: We describe a new approach to the formal solution of the raditive transfer equation for polarized light designed to deal with contact discontinuities, shock fronts, and steep gradients. We draw from concepts of computational fluid dynamics, using piecewise constant and piecewise linear reconstructions and slope limiters for the source function.

Radiative tranfer equation and formal solution

The *radiative transfer equation (RTE)* for polarized light (Unno-Rachkovsky equation) is given by

$$\frac{\mathrm{d}\mathbf{I}}{\mathrm{d}s} = -\mathbf{K}(\mathbf{I} - \mathbf{S}) \,.$$

K (the 4 × 4 absorption matrix) and S (the source function) are, in the simplest case, known functions of the coordinate *s* along the ray path. **I** = (I, Q, U, V)^T is termed as the Stokes vector. Different from the RTE for unpolarized light, the formal solution of this equation is not so straightforwardly obtained.

The *DELO method* (Rees et al. 1989), makes use of the fact that the diagonal elements of the absorption matrix are all identical, allowing to recast the equation in a form equivalent to the ordinary RTE:

$$\frac{\mathrm{d}\mathbf{I}}{\mathrm{d}\tau} = \mathbf{I} - \mathscr{S} \,,$$

where $\mathscr{S}$ is an effective source function that depends on the off-diagonal elements of **K** times the Stokes vector **I**. When linearly interpolating $\mathscr{S}$ like

$$\mathscr{S}(\tau) = [(\tau_{i+1} - \tau)\mathscr{S}_i + (\tau - \tau_i)\mathscr{S}_{i+1}]\,/\Delta\tau \,,$$

and writing the formal solution on the optical path from *i* to *i*+1,

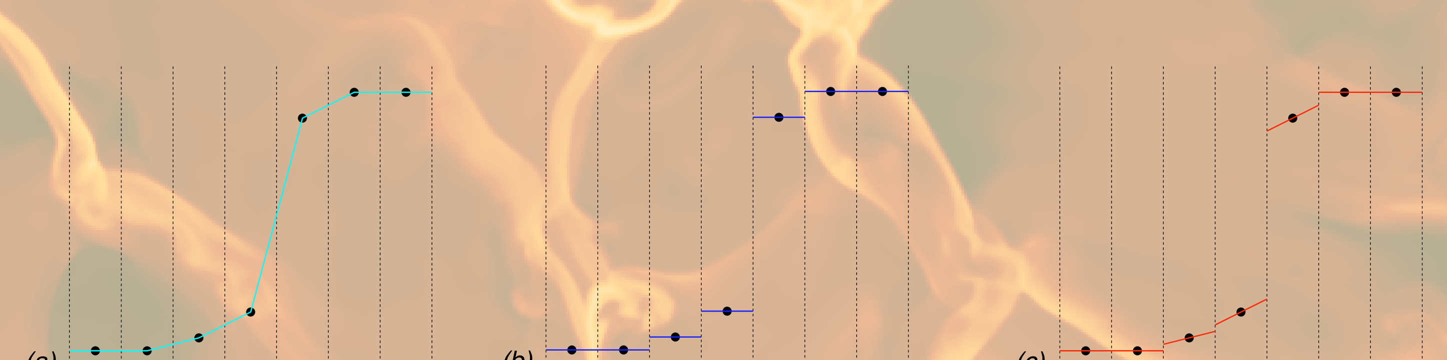
$$\mathbf{I}(\tau_i) = \mathbf{I}(\tau_{i+1})\,\mathrm{e}^{-\Delta\tau} + \int_{\tau_i}^{\tau_{i+1}} \mathrm{e}^{-(\tau-\tau_i)}\mathscr{S}(\tau)\,\mathrm{d}\tau \,,$$

one can express **I**(τ_i) in terms of **I**(τ_{i+1}).

The *method of Landi Degl'Innocenti & Landi Degl'Innocenti (1985)* dispenses with the effective source function but presumes that the absorption matrix is constant (times a linear factor). Both methods can be used to analytically integrate the RTE for polarized light on short ray paths with constant absorption matrix and simple source function given an arbitrary incident Stokes vector **I**.

Borrowing from computational fluid dynamics

In the past, the linear interpolation of the effective source function in the DELO method was replaced by quadratic interpolation (Trujillo Bueno, 2003) and by Bézier splines (Auer 2003; Štepán & Trujillo Bueno, 2013; De la Cruz Rodríguez & Piskunov, 2013) for achieving higher accuracy. Here, we propose a more simple scheme. We use the formal solution of Landi Degl'Innocenti & Landi Degl'Innocenti (1985) which uses the true source function directly but reconstruct it *piecewise continuous* only, allowing for jumps at interfaces from one discrete ray path to the next.

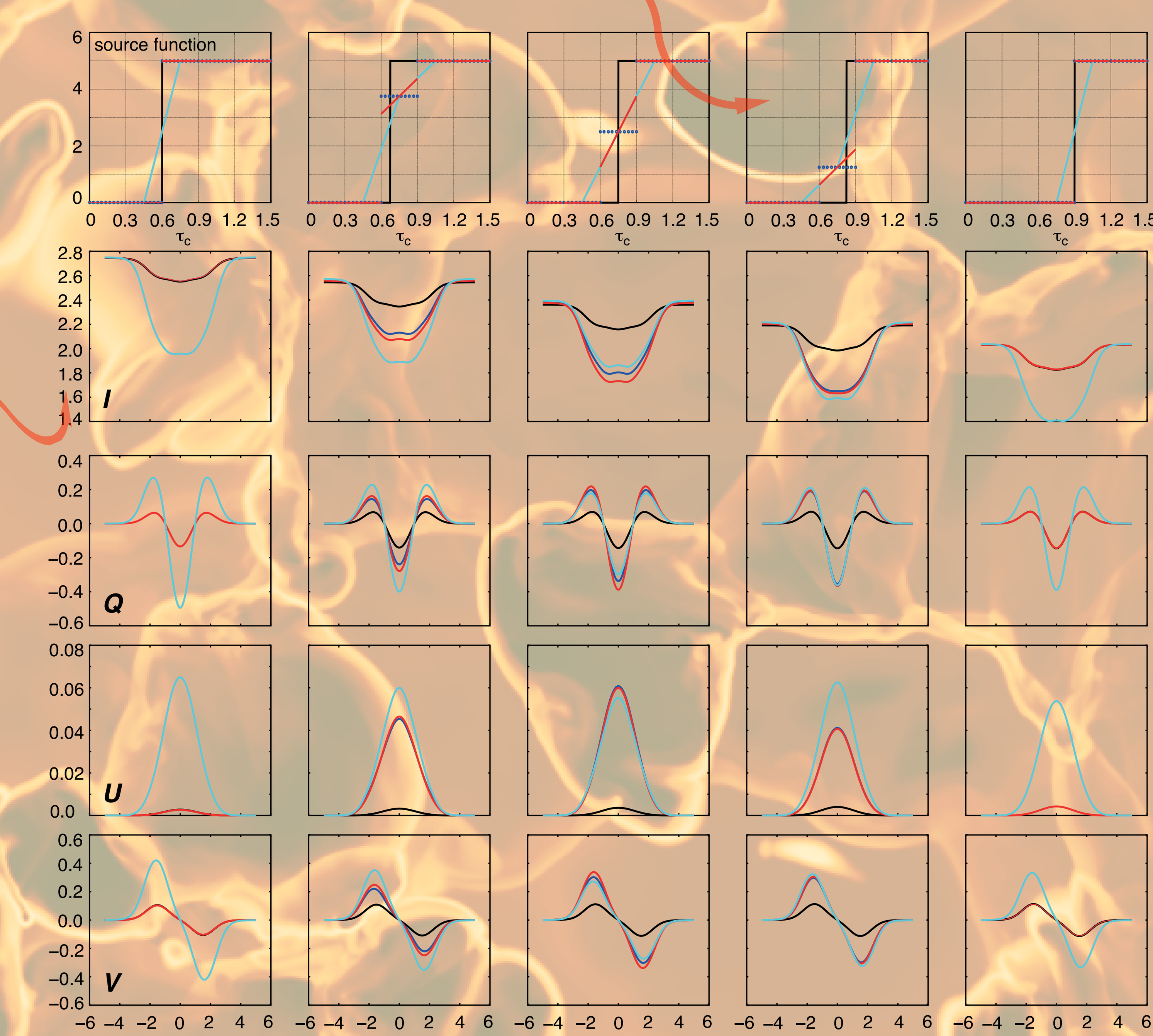


Panel (a) shows a discrete source function (black dots) which is continuously connected by linear interpolants as done with the DELO method. In panel (b) the source function is (conservatively) reconstructed by a piecewise constant function. Panel (c) shows a piecewise linear reconstruction using the minmod slope limiter. The piecewise continuous reconstructions allow for true discontinuities at grid interfaces.

First results

We consider the case in which polarization is introduced by the Zeeman effect only, i.e., by magnetic fields. For a first proof of concept, we have created an Octave program, which enables us to carry out basic RT problems in the Zeeman regime. A first test series comprises an *atmosphere with a single discontinuity* that is moved across one grid cell.

At first, the discontinuity is at τ_c= 0.6, where the source function jumps from 0 to 6 and the line opacity κ_l from 0.5 to 20. With this choice, the spectral line and continuum are confined to form close to the discontinuity for obtaining maximal impact from it.



The *first row* shows the original source function (black lines) with a discontinuity that moves from τ_c= 0.6 to τ_c= 0.9 across one grid cell from left to right, respectively. The cyan lines are the linear interpolation of the discrete representation of the original source function and can be considered representatives of the DELO method. The blue dotted lines correspond to the piecewise constant reconstruction (Godunov), and the red lines to the piecewise linear reconstruction with minmod slope limiter (van Leer).

The *corresponding Stokes profiles* (I,Q,U,V)^T are shown below the respective source function, where the black curves represent the exact solution. In the first and last column, when the discontinuity coincides with a grid interface, the piecewise continuous reconstructions reproduce the solution exactly. In between, the new method performs less spectacular. In fact, when the discontinuity is in the middle of the cell, the traditional method performs even slightly better.

References

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Background image: Shock waves in the solar chromosphere from a numerical simulation carried out at CSCS. Courtesy of Flavio Calvo (IRSOL)